

General Capacity for Deterministic Dissemination in Wireless Ad Hoc Networks

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Abstract—In this paper, we study capacity scaling laws of the deterministic dissemination (DD) in random wireless networks under the generalized physical model (GphyM). This is truly not a new topic. Our motivation to readdress this issue is two-fold: Firstly, we aim to propose a more general result to unify the network capacity for general homogeneous random models by investigating the impacts of different parameters of the system on the network capacity. Secondly, we target to close the open gaps between the upper and the lower bounds on the network capacity in the literature. The generality of this work lies in three aspects: (1) We study the homogeneous random network of a general node density $\lambda \in [1, n]$, rather than either random dense network (RDN, $\lambda = n$) or random extended network (REN, $\lambda = 1$) as in the literature. (2) We address the general deterministic dissemination sessions, *i.e.*, the general multicast sessions, which unify the capacities for unicast and broadcast sessions by setting the number of destinations for each session as a general value $n_d \in [1, n]$. (3) We allow the number of sessions to change in the range $n_s \in (1, n]$, instead of assuming that $n_s = \Theta(n)$ as in the literature. We derive the general upper bounds on the capacity for the arbitrary case of (λ, n_d, n_s) by introducing the Poisson Boolean model of continuum percolation, and prove that they are tight according to the existing general lower bounds constructed in the literature.

Index Terms—Network Capacity, Scaling Laws, Deterministic Dissemination, Random Wireless Networks, Percolation Theory

I. INTRODUCTION

This work falls within the scope of the issue of capacity scaling laws for wireless networks, initiated by Gupta and Kumar [2], *i.e.*, the scaling of network performance in the limit when the network gets large, [3]. The main advantage of studying scaling laws is to highlight qualitative and architectural properties of the system without considering too many details [2], [3]. The network capacity depends directly on the type of traffic sessions of interest. Generally, the traffic sessions in wireless networks can be classified into two broad types: *data dissemination*, where a session has only one source, and *data gathering*, where a session intends to transmit data from its multiple sources to a relatively small number of destinations; on the other hand, according to the property

of destination selection schemes, they can also be divided into the following two types: *deterministic session*, where the selection of destination(s) are/is determined beforehand, and *opportunistic session*, where the destination(s) are/is opportunistically chosen during the transmitting procedure. Based on those classifications, the typical session patterns can be located as shown in Table. I.

In this work, we focus on dissemination sessions that can be usually represented by a triple dimensional vector (n, n_c, n_d) with $1 \leq n_d \leq n_c \leq n - 1$. These parameters are defined by the following: The node set of network, say $\mathcal{V} := \mathcal{V}(n)$, comprises n nodes; the cardinality of source set $\mathcal{S} \subseteq \mathcal{V}$ is $|\mathcal{S}| = n_s$; during the process of dissemination with source $v_i \in \mathcal{S}$, n_c nodes are randomly chosen to compose a *candidate set*, denoted by \mathcal{C}_i , and the session is completed when data are transmitted to a subset $\mathcal{D}_i \subseteq \mathcal{C}_i$, called *destination set*, where $|\mathcal{D}_i| = n_d \leq n_c$. Obviously, when $n_d \equiv n_c$, the dissemination is specified into a *deterministic dissemination*, *i.e.*, the so-called *general multicast session*. Please see the illustration in Fig.1.

The purpose of this paper is to investigate the capacity of wireless networks where $n_s : (1, n]^1$ general multicast sessions, denoted by (n, n_d, n_d) with $n_d : [1, n]$, run simultaneously. In the research of *networking-theoretic* capacity scaling laws [3], the unicast and broadcast sessions can be usually regarded as two special cases of general multicast sessions according to the number of destinations for each session. Usually, any proposed multicast capacity could be specialized into the unicast and broadcast capacities by letting $n_d = 1$ and $n_d = n$, respectively. This principle often applies in the literatures [4]–[9].

Most of the existing results differ from each other due to the diversity of adopted analytical models and assumptions. Besides session patterns introduced above, there are two typical models in terms of scaling patterns that are adopted in the literature: *random extended network* (REN), where the node density is fixed to a constant [6], [8], [10], [11], and *random dense network* (RDN), where the node density increases linearly with the number of nodes [2], [5], [12]–[14]. In [13], Shakkottai *et al.* derived the multicast capacity of RDN for a specific case that $n_s = n^\epsilon$ and $n_s \cdot n_d = \Theta(n)$, where $\epsilon \in (0, 1]$. They showed that such per-session multicast

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¹We use the term $f(n) : [\phi_1(n), \phi_2(n)]$ to represent $f(n) = \Omega(\phi_1(n))$ and $f(n) = O(\phi_2(n))$; and use $f(n) : \langle \phi_1(n), \phi_2(n) \rangle$ to represent $f(n) = \omega(\phi_1(n))$ and $f(n) = o(\phi_2(n))$.

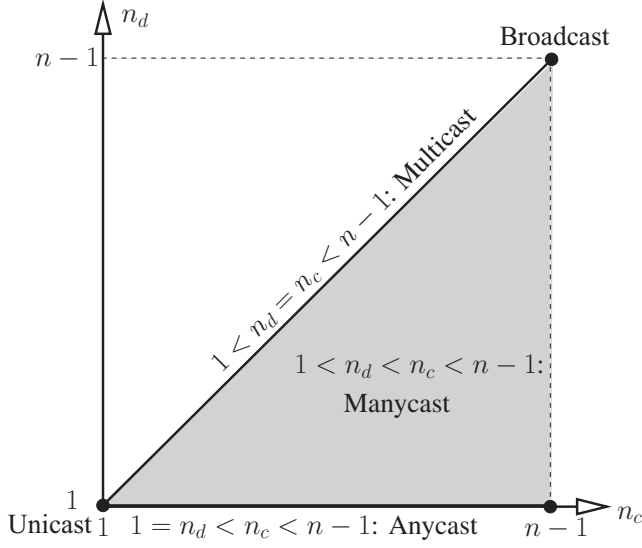


Fig. 1. General Dissemination Sessions. Here, m is the size of the specified group for a source, where $1 \leq m \leq n-1$; d is the number of desired destinations of the source, where $1 \leq d \leq m$.

capacity under the protocol model is at most of $O(\frac{1}{\sqrt{n_s \log n}})$. To achieve the upper bound, they proposed a simple and novel routing architecture, called the *multicast comb*, to transfer multicast data in the network. A more general result, in terms of n_s and n_d , was proposed by Li *et al.* in [4]. They showed that when $n_s = \Omega(\log n_d \cdot \sqrt{n \log n / n_d})$, the per-session multicast capacity for RDN under the protocol model is of $\Theta(\frac{1}{n_s} \sqrt{\frac{n}{n_d \log n}})$ if $n_d = O(\frac{n}{\log n})$, and is of $\Theta(1/n_s)$ if $n_d = \Omega(n/\log n)$. After that, Keshavarz-Haddad *et al.* [5] derived the multicast capacity for RDN under the *generalized physical model* [15] by designing new multicast schemes and computing the upper bounds. A *gap* remains open between the upper and the lower bounds in the regime $n_d : [n/(\log n)^3, n/\log n]$ as illustrated in Fig.2(a). For multicast capacity of REN under the generalized physical model, Li *et al.* [6] derived a lower bound as $\Omega(\frac{\sqrt{n}}{n_s \sqrt{n_d}})$ for the case that $n_s = \Omega(n^{1/2+\epsilon})$ and $n_d = O(n/(\log n)^{2\alpha+6})$. Recently, Wang *et al.* [8] devised the specific multicast schemes and derived the multicast throughput for all cases $n_s : [1, n]$ and $n_d : [1, n]$. Under the assumption that $n_s = \Theta(n)$, their lower bounds are specialized into those in Equation (4). They also derived an upper bound for the case that $n_s = \Theta(n)$, as in Equation (5). An obvious *gap* exists between the upper and the lower bounds in the regime $n_d : [n/(\log n)^{\alpha+1}, n/\log n]$ (Please see the illustration in Fig.2(b)). To the best of our knowledge, this is the latest results on general multicast capacity for static REN without considering the impacts of node mobility [7], [16] or advanced physical communication technology [17], [18]. *Closing* the remaining *gaps* is one of the motivations of this paper.

Both REN and RDN are extreme cases for a random network consisting of n nodes in terms of the node density

TABLE I
TYPICAL SESSION PATTERNS

	Deterministic Session	Opportunistic Session
Dissemination /Single-Source	Unicast Broadcast Multicast	Anycast ... Multicast
Gathering /Multiple-Sources	Data Collection ... ConvergeCast (Many-to-One)	Undefined (to the best of our knowledge)

λ . So the characterization of two particular models does not suffice to develop a comprehensive understanding of wireless networks, although they are representative models to some extent, [3]. Hence, in this paper, we comprehensively consider the network with a general node density $\lambda : [1, n]$, rather than only the cases $\lambda = 1$ (REN) and $\lambda = n$ (RDN), which can offer complete and deep insights about the scaling laws for wireless networks. *Unearthing the nature* of general scaling is another motivation of this work.

In conclusion, we aim to examine the capacity scaling laws of general wireless networks, where the generality lies in three aspects: (1) a general node density, $\lambda : [1, n]$; (2) a general number of receivers, $n_d : [1, n]$; (3) a general number of sessions, $n_s : [1, n]$. For such general multicast capacity of general wireless networks, we have computed the lower bounds under the generalized physical model in [1]. More specifically, we build routing backbones of two levels: *highways* and *arterial roads*. Furthermore, arterial roads (ARs) have two subclasses, *i.e.*, *ordinary arterial roads* (O-ARs) and *parallel arterial roads* (P-ARs). Note that the highways are the same as those in [5], [6], [8], [10], but the ARs are different from the *second-class highways* (SHs) in [8]. Recall that in the SH system of [8], there are two types of SHs: *odd* SHs and *even* SHs. The bottleneck of the whole routing could happen in the switching phase between the odd and even SHs. There is no such a bottleneck in the current AR system, which can improve the multicast throughput for some regimes of n_s and n_d . Based on the highways, O-ARs and P-ARs, we design four routing schemes. By exploiting the theory of *maximum occupancy*, we derive the optimal multicast throughput and scheme according to different ranges of λ , n_d , and n_s .

Major contributions of this paper can be summarized as follows:

▷ For deriving the upper bounds on multicast capacity, we introduce the Poisson Boolean model of continuum percolation [19] (not Poisson bond percolation model [10]), which, to the best of our knowledge, is not used in previous studies on upper bounds of network capacity. Based on the argument of *giant cluster* (component) in the Poisson boolean percolation model, we can divide the communications under any multicast routing scheme into two parts, *i.e.*, communications inside and outside the giant component. Obviously, the network throughput must be determined by the bottleneck of two parts. We give a

general formula to compute upper bounds on the capacity.

▷ For the case that $n_s = \Theta(n)$ and $\lambda = n$ (or $\lambda = 1$), i.e., RDN and REN, due to the limitations of adopted analytical methods, the previous works [5], [8] have not derived the tight bounds on multicast capacity under the generalized physical model. By applying our general results to these special cases, we close those gaps.

The rest of the paper is organized as follows. The system model is formulated in Section II. We present and discuss the main results in Section III. In Section IV, we make preparations for the analysis. We derive the upper bounds on the capacity in Section VI. For completeness, we include the derivation of lower bounds from [1] in Appendix A. We draw some conclusions in Section VII.

II. SYSTEM MODEL

A. Random Scaling Model

We construct a random network with node density λ , denoted by $\mathcal{N}(\lambda, n)$, by placing wireless nodes randomly into a square region $\mathcal{R}(\lambda, n) = [0, \sqrt{A}]^2$ according to a Poisson point process with density λ , where $A = n/\lambda$. When λ is set to be 1 (or n), our model corresponds to *random extended network* (REN) (or *random dense network* (RDN)). According to Chebyshev's inequality, we get that the number of nodes in $\mathcal{A}(a^2)$ is within $((1 - \epsilon)n, (1 + \epsilon)n)$ with high probability, where $\epsilon > 0$ is an arbitrarily small constant. To simplify the description, we assume that the number of nodes is exactly n , without changing our results in the sense of order, [8], [10], [11]. We are mainly concerned with the events that occur inside these squares with high probability (w.h.p.); that is, with probability approaching one as $n \rightarrow \infty$.

B. Session Patterns

In wireless networks, there are two broad types of session patterns: *information dissemination* and *information gathering*. The former is the interest of this paper. Generally, dissemination sessions can be further divided into two categories: *deterministic dissemination*, in which the destination(s) of a message is (are) determined when it is generated at a source, such as unicast, broadcast, and multicast, and *opportunistic dissemination*, such as anycast [20], [21], and manycast [22] sessions, in which the destination(s) of a message is (are) opportunistically chosen and both the paths to the group member(s) and the destination(s) can change dynamically according to the network condition, such as the node movement situation.

In this work, we focus on the general multicast sessions, including unicast, broadcast and multicast sessions. We adopt a similar construction procedure to the one in [8]. To generate the k -th ($1 \leq k \leq n_s$) multicast session, with source $v_{S,k} \in \mathcal{S}$, denoted by $\mathcal{M}_{S,k}$, n_d points $p_{S,k,i}$ ($1 \leq i \leq n_d$, and $1 \leq n_d \leq n - 1$) are randomly and independently chosen from the deployment region $\mathcal{R}(\lambda, n)$. Denote the set of these n_d points by $\hat{\mathcal{P}}_{S,k} = \{p_{S,k,1}, p_{S,k,2}, \dots, p_{S,k,n_d}\}$. Let $v_{S,k,i}$ be the

nearest ad hoc node from $p_{S,k,i}$ (ties are broken randomly). In $\mathcal{M}_{S,k}$, the node $v_{S,k}$, serving as a source, intends to deliver data to n_d destinations $\mathcal{D}_{S,k} = \{v_{S,k,1}, v_{S,k,2}, \dots, v_{S,k,n_d}\}$ at an arbitrary data rate $\lambda_{S,k}$. Let $\mathcal{U}_{S,k} = \{v_{S,k}\} \cup \mathcal{D}_{S,k}$ be the *spanning set* of nodes for the multicast session $\mathcal{M}_{S,k}$. Please see the illustration in Fig.3.

C. Communication Model

Generally, there are three types of communication (interference) models: the *protocol model* [2], *physical model* [2] and *generalized physical model* [15] (along with the name Gaussian Channel model, [6]). We adopt the generalized physical model since it is more realistic than the other two [6], [10], [14], [15].

Let \mathcal{K}_t denote a *scheduling set* of links in which all links can be scheduled simultaneously in time slot t .

Definition 1: Under the generalized physical model, when a scheduling set \mathcal{K}_t is scheduled, the rate of a link $\langle u, v \rangle \in \mathcal{K}_t$ is achieved at

$$R_{u,v;t} = B \times \mathbf{1} \cdot \{\langle u, v \rangle \in \mathcal{K}_t\} \times \log(1 + \text{SINR}_{u,v;t}), \quad (1)$$

where $\text{SINR}_{u,v;t} = \frac{P \cdot \ell(|\mathbf{x}_u - \mathbf{x}_v|)}{N_0 + \sum_{\langle i, j \rangle \in \mathcal{K}_t / \langle u, v \rangle} P \cdot \ell(|\mathbf{x}_i - \mathbf{x}_j|)}$; \mathbf{x}_u denotes the position of node u , $|\mathbf{x}_u - \mathbf{x}_v|$ represents the Euclidean distance between node u and node v ; $\ell(\cdot)$ denotes the power attenuation function that is assumed to depend only on the distance between the transmitter and the receiver [2], [6], [10], [23]; $\ell(|\cdot|) := |\cdot|^{-\alpha}$ for dense scaling networks, and $\ell(|\cdot|) := \min\{1, |\cdot|^{-\alpha}\}$ for extended scaling networks [10].

III. MAIN RESULTS

We mainly derive the upper bounds on the general multicast capacity of random ad hoc networks.

A. General Upper Bounds

Theorem 1: The multicast capacity for random network $\mathcal{N}(\lambda, n)$ is at most

$$\bar{\Lambda}(\lambda, n) = \max_{l_c \in \mathcal{L}_c} \left\{ \min \left\{ \frac{\min\{1, l_c^{-\alpha}\}}{\mathbf{L}(n_s, \frac{\sqrt{n}}{l_c \sqrt{n_d \lambda}})}, \frac{\min\{1, (\frac{\lambda}{\log n})^{\frac{\alpha}{2}}\}}{\mathbf{L}(n_s, \frac{n \cdot \sqrt{\lambda} \cdot l_c}{n_d \cdot \sqrt{\log n}})} \right\} \right\},$$

where $\mathcal{L}_c = [1/\sqrt{\lambda}, \sqrt{\log n/\lambda}]$, and $\mathbf{L}(m, n)$ is defined in Table II.

B. Tight Capacity Bounds

In [1], the general lower bounds have been provided by designing some strategies.

Lemma 1 ([1]): The general multicast throughput for random network $\mathcal{N}(\lambda, n)$ can be achieved as

$$\underline{\Lambda}(\lambda, n) = \max\{\Lambda_o(\lambda, n), \Lambda_p(\lambda, n), \Lambda_{o\&h}(\lambda, n), \Lambda_{p\&h}(\lambda, n)\},$$

where $\Lambda_o(\lambda, n), \Lambda_p(\lambda, n), \Lambda_{o\&h}(\lambda, n), \Lambda_{p\&h}(\lambda, n)$ are defined in Table II.

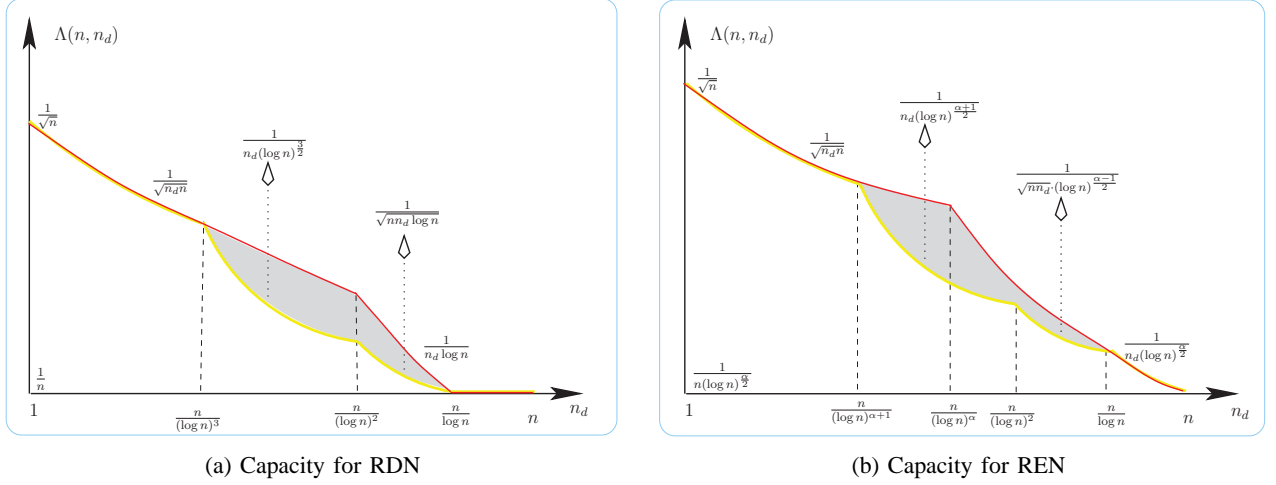


Fig. 2. Results on General Multicast Capacity. Obvious *gaps* exist between the upper and the lower bounds in the regimes $n_d : [n/(\log n)^3, n/\log n]$ for RDN and $n_d : [n/(\log n)^{\alpha+1}, n/\log n]$ for REN, illustrated by the shaded regions.

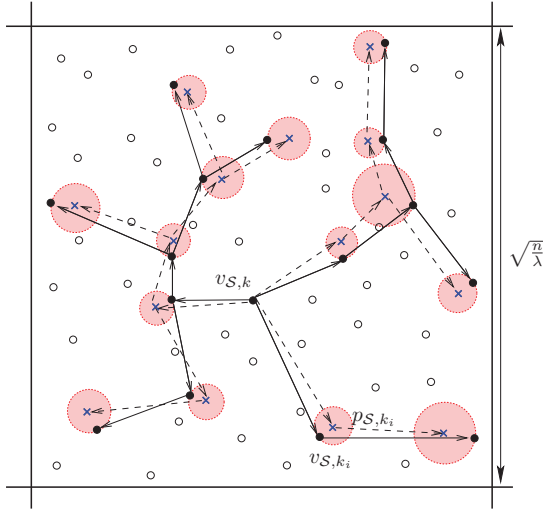


Fig. 3. Multicast session $\mathcal{M}_{S,k}$, [8]. The tree consisting of solid lines represents the Euclidean minimum spanning tree (EMST) over $\mathcal{U}_{S,k} = \{v_{S,k_i} \mid 0 \leq i \leq n_d\}$, denoted by $\text{EMST}(\mathcal{U}_{S,k})$, where v_{S,k_0} is $v_{S,k}$. The tree consisting of dashed lines represents an Euclidean spanning tree (EST) over $\mathcal{P}_{S,k} = \{p_{S,k_i} \mid 0 \leq i \leq n_d\}$, denoted by $\text{EST}_0(\mathcal{P}_{S,k})$, where p_{S,k_0} is $v_{S,k}$, and for any $0 \leq i, j \leq n_d$, link $(p_{S,k_i} \rightarrow p_{S,k_j}) \in \text{EST}_0(\mathcal{P}_{S,k})$ if and only if link $(v_{S,k_i} \rightarrow v_{S,k_j}) \in \text{EMST}(\mathcal{U}_{S,k})$.

We specialize the general results from Theorem 1 and Lemma 1 to the cases that $\lambda = n$ and $\lambda = 1$, corresponding to the RDN and REN. Following a common assumption in most existing works, *i.e.*, $n_s = \Theta(n)$, we show that for both RDN and REN our results give the first tight bounds on multicast capacity over the whole regime $n_d : [1, n]$.

1) *Random Dense Networks*: In Theorem 1, $\bar{\Lambda}(n, n)$, *i.e.*, the upper bound on the capacity achieves its maximum value by choosing $l_c = \Theta(\frac{1}{\sqrt{n}})$ when $n_d = O(n/(\log n)^2)$; and also achieves its maximum value by choosing $l_c = \Theta(\sqrt{\log n}/\sqrt{n})$ when $n_d = \Omega(n/(\log n)^2)$. Specifically, the multicast capacity

is *at most* of order

$$\begin{cases} \Theta(\frac{1}{\sqrt{n_d n}}) & \text{when } n_d : [1, \frac{n}{(\log n)^3}] \\ \Theta(\frac{1}{n_d (\log n)^{\frac{\alpha}{2}}}) & \text{when } n_d : [\frac{n}{(\log n)^3}, \frac{n}{\log n}] \\ \Theta(\frac{1}{\sqrt{n_d \log n}}) & \text{when } n_d : [\frac{n}{(\log n)^2}, \frac{n}{\log n}] \\ \Theta(\frac{1}{n}) & \text{when } n_d : [\frac{n}{\log n}, n] \end{cases} \quad (2)$$

This result is *exciting*, because the multicast throughput as in Equation (2) had been proven to be achievable by Keshavarz-Haddad *et al.* in [5]. Moreover, they derived an upper bound as

$$\begin{cases} O(\frac{1}{\sqrt{n_d n}}) & \text{when } n_d : [1, \frac{n}{(\log n)^2}] \\ O(\frac{1}{n_d \log n}) & \text{when } n_d : [\frac{n}{(\log n)^2}, \frac{n}{\log n}] \\ O(\frac{1}{n}) & \text{when } n_d : [\frac{n}{\log n}, n] \end{cases} \quad (3)$$

It is clear that there is a gap between the upper and the lower bounds in the regime $n_d : (\frac{n}{(\log n)^3}, \frac{n}{\log n})$, as illustrated in Fig.2(a). In this work, we *close* this gap. Moreover, by Lemma 1, this optimal throughput in Equation (2) can also be achieved by using our schemes \mathbb{M}_0 cooperatively and $\mathbb{M}_{0\&h}$ that are defined in Table A.1 in Appendix A.

2) *Random Extended Networks*: In Theorem 1, $\bar{\Lambda}(1, n)$ achieves its maximum value by letting $l_c = \Theta(1)$ when $n_d = O(n/(\log n)^2)$; and achieves its maximum value by letting $l_c = \Theta(\sqrt{\log n})$ when $n_d = \Omega(n/(\log n)^2)$. Specifically, the multicast capacity is *at most* of order

$$\begin{cases} \Theta(\frac{1}{\sqrt{n_d n}}) & \text{when } n_d : [1, \frac{n}{(\log n)^{\alpha+1}}] \\ \Theta(\frac{1}{n_d (\log n)^{\frac{\alpha+1}{2}}}) & \text{when } n_d : [\frac{n}{(\log n)^{\alpha+1}}, \frac{n}{(\log n)^2}] \\ \Theta(\frac{1}{\sqrt{n_d \log n}}) & \text{when } n_d : [\frac{n}{(\log n)^2}, \frac{n}{\log n}] \\ \Theta(\frac{1}{n_d (\log n)^{\frac{\alpha}{2}}}) & \text{when } n_d : [\frac{n}{\log n}, n] \end{cases} \quad (4)$$

Also, such multicast throughput had been achieved by the schemes in [8], and the upper bounds were proposed as:

$$\begin{cases} O(\frac{1}{\sqrt{n_d n}}) & \text{when } n_d : [1, \frac{n}{(\log n)^\alpha}] \\ O(\frac{1}{n_d (\log n)^{\frac{\alpha}{2}}}) & \text{when } n_d : [\frac{n}{(\log n)^\alpha}, n] \end{cases} \quad (5)$$

TABLE II
DEFINED FUNCTIONS AND PARAMETERS.

Functions	Definitions
$\mathbf{L}(m, n)$	$\begin{cases} \Theta\left(\frac{\log n}{\log \frac{n}{m}}\right) & \text{when } m : [1, \frac{n}{\text{polylog}(n)}) \\ \Theta\left(\frac{\log n}{\log \frac{n \log n}{m}}\right) & \text{when } m : [\frac{n}{\text{polylog}(n)}, n \log n] \\ \Theta\left(\frac{m}{n}\right) & \text{when } m = \Omega(n \log n) \end{cases}$
$\mathbf{R}_{\text{O-AR}}(\lambda, n)$	$\begin{cases} \Theta\left(\frac{\lambda^{\frac{\alpha}{2}}}{(\log n)^{\frac{\alpha}{2}}}\right) & \text{when } \lambda : [1, \log n] \\ \Theta(1) & \text{when } \lambda : [\log n, n] \end{cases}$
$\mathbf{R}_{\text{P-AR}}(\lambda, n)$	$\begin{cases} \Theta\left(\frac{\lambda^{\frac{\alpha}{2}}}{(\log n)^{\frac{\alpha}{2}}}\right) & \text{when } \lambda : [1, (\log n)^{1-\frac{2}{\alpha}}] \\ \Theta\left(\frac{1}{\log n}\right) & \text{when } \lambda : [(\log n)^{1-\frac{2}{\alpha}}, n] \end{cases}$
\mathbf{p}_o	$\begin{cases} \Theta\left(\sqrt{\frac{n_d \log n}{n}}\right) & \text{when } n_d = O\left(\frac{n}{\log n}\right) \\ \Theta(1) & \text{when } n_d = \Omega\left(\frac{n}{\log n}\right) \end{cases}$
\mathbf{p}_p	$\begin{cases} \Theta\left(\frac{\sqrt{n_d}}{\sqrt{n} \log n}\right) & \text{when } n_d : [1, \frac{n}{\log n}] \\ \Theta\left(\frac{n_d}{n}\right) & \text{when } n_d : [\frac{n}{\log n}, n] \end{cases}$
$\mathbf{p}_{\text{oh}, \text{O-AR}}$	$\begin{cases} \Theta\left(\frac{n_d \cdot (\log n)^{3/2}}{n}\right) & \text{when } n_d : [1, \frac{n}{(\log n)^{3/2}}] \\ \Theta(1) & \text{when } n_d : [\frac{n}{(\log n)^{3/2}}, n] \end{cases}$
$\mathbf{p}_{\text{oh}, \text{H}}, \mathbf{p}_{\text{ph}, \text{H}}$	$\begin{cases} \Theta\left(\sqrt{\frac{n_d}{n}}\right) & \text{when } n_d : [1, \frac{n}{(\log n)^2}] \\ \Theta\left(\frac{n_d \log n}{n}\right) & \text{when } n_d : [\frac{n}{(\log n)^2}, \frac{n}{\log n}] \\ \Theta(1) & \text{when } n_d : [\frac{n}{\log n}, n] \end{cases}$
$\mathbf{p}_{\text{ph}, \text{P-AR}}$	$\begin{cases} \Theta\left(\frac{n_d \cdot \sqrt{\log n}}{n}\right) & \text{when } n_d : [1, \frac{n}{\sqrt{\log n}}] \\ \Theta(1) & \text{when } n_d : [\frac{n}{\sqrt{\log n}}, n] \end{cases}$
$\Lambda_o(\lambda, n)$	$\mathbf{R}_{\text{O-AR}}(\lambda, n) / \mathbf{L}(n_s, \frac{1}{\mathbf{p}_o})$
$\Lambda_p(\lambda, n)$	$\mathbf{R}_{\text{P-AR}}(\lambda, n) / \mathbf{L}(n_s, \frac{1}{\mathbf{p}_p})$
$\Lambda_{o\&h}(\lambda, n)$	$\min \left\{ \frac{\mathbf{R}_{\text{O-AR}}(\lambda, n)}{\mathbf{L}(n_s, \frac{1}{\mathbf{p}_{\text{oh}, \text{O-AR}}})}, \frac{1}{\mathbf{L}(n_s, \frac{1}{\mathbf{p}_{\text{oh}, \text{H}}})} \right\}$
$\Lambda_{p\&h}(\lambda, n)$	$\min \left\{ \frac{\mathbf{R}_{\text{P-AR}}(\lambda, n)}{\mathbf{L}(n_s, \frac{1}{\mathbf{p}_{\text{ph}, \text{P-AR}}})}, \frac{1}{\mathbf{L}(n_s, \frac{1}{\mathbf{p}_{\text{ph}, \text{H}}})} \right\}$

As illustrated in Fig.2(b), we close the gap between the upper and the lower bounds in the regime $n_d : [\frac{n}{(\log n)^{\alpha+1}}, \frac{n}{\log n}]$. In addition, by Lemma 1, this optimal throughput in Equation (4) can be equally achieved by using our schemes \mathbb{M}_p and $\mathbb{M}_{p\&h}$ cooperatively that are defined in Table A.1 in Appendix A.

IV. TECHNICAL PREPARATIONS

A. Maximum Occupancy

We use the results in *maximum occupancy theory* to derive the lower bounds of the multicast throughput. Now we introduce the following result from [24], [25] and [26].

Lemma 2: Let $\mathbf{L}(m, n)$ be the random variable that counts the maximum number of balls in any bin, if we throw m balls independently and uniformly at random into n bins. Then, it

holds that *w.h.p.*,

$$\mathbf{L}(m, n) = \begin{cases} \Theta\left(\frac{\log n}{\log \frac{n}{m}}\right) & \text{when } m : [1, \frac{n}{\text{polylog}(n)}) \\ \Theta\left(\frac{\log n}{\log \frac{n \log n}{m}}\right) & \text{when } m : [\frac{n}{\text{polylog}(n)}, n \log n] \\ \Theta\left(\frac{m}{n}\right) & \text{when } m = \Omega(n \log n) \end{cases}$$

B. Network Throughput by Occupancy Theory

We give a technical lemma as a basic argument of the analysis of network capacity.

Lemma 3: Given a multicast scheme \mathbb{M} , for any link initiating from a node u , say uv , if it can sustain a rate of $\mathbf{R}(\lambda, n)$, and any multicast session shares the bandwidth of link uv with the probability of \mathbf{p} , then the throughput along link uv is of order $\Theta(\Lambda(\lambda, n))$, where $\Lambda(\lambda, n) = \frac{\mathbf{R}(\lambda, n)}{\mathbf{L}(n_s, 1/\mathbf{p})}$.

C. The Tail of Poisson Trials

Lemma 4 ([27]): Let X_1, X_2, \dots, X_n be independent Poisson trials such that, for $1 \leq i \leq n$, $\Pr[X_i = 1] = p_i$, where $0 < p_i < 1$. Then, for $X = \sum_{i=1}^n X_i$, $\mu = \mathbb{E}(X) = \sum_{i=1}^n p_i$, and any $\delta > 0$,

$$\Pr[X > (1 + \delta)\mu] < \left[\frac{e^\delta}{(1 + \delta)^{1+\delta}} \right]^\mu.$$

D. Euclidean Spanning Tree

Lemma 5 ([8]): If X_i , $1 \leq i \leq \infty$, are uniformly distributed on $[0, a]^d$, for a set $\mathcal{U}(n) = \{X_1, X_2, \dots, X_n\}$, denote its Euclidean minimum spanning tree (EMST) by $\text{EMST}(\mathcal{U}(n))$. Under such deployment model, build $K(n)$ sets, denoted by $\mathcal{U}_1(n), \mathcal{U}_2(n), \dots, \mathcal{U}_{K(n)}(n)$, it holds that

$$\Pr \left[\lim_{n \rightarrow \infty} \frac{\sum_{k=1}^{K(n)} \|\text{EMST}(\mathcal{U}_k(n))\|}{K(n) \cdot a \cdot n^{1-\frac{1}{d}}} = \nu(d) \right] = 1. \quad (6)$$

This lemma can be straightforwardly proven according to Theorem 2 of [28]. Please see the detailed proof of Lemma D in the appendices of [8].

For any n_s general multicast sessions constructed by the method in Section II-B, by a similar procedure to Lemma 7 of [8], we have,

Lemma 6: For all multicast sessions $\mathcal{M}_{\mathcal{S},k}$ ($1 \leq k \leq n_s$), it holds that for $n_d = o(\frac{n}{\log n})$,

$$\sum_{k=1}^{n_s} \|\text{EMST}(\mathcal{D}_{\mathcal{S},k})\| = \Omega(n_s \cdot \sqrt{n_d \cdot n}),$$

where $\text{EMST}(\mathcal{D}_{\mathcal{S},k})$ denotes the Euclidean minimum spanning tree (EMST) over the destination set $\mathcal{D}_{\mathcal{S},k}$.

Note that the session construction in this work is different from that in [8], and Lemma 6 only gives a result on $\sum_{k=1}^{n_s} \|\text{EMST}(\mathcal{D}_{\mathcal{S},k})\|$ instead of $\sum_{k=1}^{n_s} \|\text{EMST}(\mathcal{M}_{\mathcal{S},k})\|$, where $\text{EMST}(\mathcal{M}_{\mathcal{S},k})$ denotes the Euclidean minimum spanning tree (EMST) over the spanning set $\mathcal{U}_{\mathcal{S},k}$. Since it holds that $\|\text{EMST}(\mathcal{M}_{\mathcal{S},k})\| \geq \|\text{EMST}(\mathcal{D}_{\mathcal{S},k})\|$, we can obtain the following corollary.

Corollary 1: For all multicast sessions $\mathcal{M}_{\mathcal{S},k}$ ($1 \leq k \leq n_s$), it holds that for $n_d = o(\frac{n}{\log n})$,

$$\sum_{k=1}^{n_s} \|\text{EMST}(\mathcal{M}_{\mathcal{S},k})\| = \Omega(n_s \cdot \sqrt{n_d \cdot n}).$$

V. NETWORK TOPOLOGY UNDER FEASIBLE ROUTINGS

We introduce the Poisson Boolean percolation model to make preparations for computing the upper bounds on the general multicast capacity.

A. Poisson Boolean Percolation Model

In a 2-dimensional Poisson Boolean model $\mathbb{B}(\lambda, r)$ [19], nodes are distributed in \mathbb{R}^2 according to a p.p of intensity λ . Each node is associated with a closed disk of radius $r/2$. Two disks are *directly connected* if they overlap. Two disks are *connected* if there exist a sequence of directly connected disks between them. Define a *cluster* as a set of disks in which any two disks are connected. Denote the set of all clusters by $\mathcal{C}(\lambda, r)$. Let $|\mathcal{C}_i|$ denote the number of disks in a cluster $\mathcal{C}_i \in \mathcal{C}(\lambda, r)$. We can associate $\mathbb{B}(\lambda, r)$ with a graph $\mathcal{G}(\lambda, r)$, called an *associated graph*, by associating a vertex with each node in $\mathbb{B}(\lambda, r)$ and an edge with each direct connection in $\mathbb{B}(\lambda, r)$. Two models $\mathbb{B}(\lambda, r)$ and $\mathbb{B}(\lambda_0, r_0)$ lead to the same associated graph, namely $\mathcal{G}(\lambda, r) = \mathcal{G}(\lambda_0, r_0)$ if $\lambda_0 r_0^2 = \lambda r^2$. Then, the graph properties of $\mathbb{B}(\lambda, r)$ only depend on the parameter λr^2 , [29]. Let \mathcal{C} denote the cluster containing the given node, the percolation probability is thus defined as $\Pr_{\lambda, r}[|\mathcal{C}| = \infty]$. We call γ_c the *critical percolation threshold* of Poisson Boolean model in \mathbb{R}^2 when

$$\gamma_c = \sup\{\gamma := \lambda \pi r^2 \mid \Pr_{\lambda, r}[|\mathcal{C}| = \infty] = 0\}.$$

The exact value of γ_c is still open. The analytical results show that it is within the range $(0.7698\pi, 3.372\pi)$ [19], [30]. In terms of the value of $\gamma = \lambda \pi r^2$, the *subcritical phase* and *supercritical phase* can be defined, which correspond to the cases when $\gamma < \gamma_c$ and $\gamma > \gamma_c$, respectively. The following lemma will be used in our analysis.

Lemma 7 ([19], [31]): For a Poisson Boolean model $\mathbb{B}(\lambda, r)$ in \mathbb{R}^2 , there exists a value γ_c in a square region $\mathcal{R}(\lambda, n) = [0, \sqrt{n/\lambda}]^2$, as $n \rightarrow \infty$:

- if $\gamma = \lambda \pi r^2 < \gamma_c$, i.e., in the subcritical phase [19], it holds that

$$\Pr[\sup\{|\mathcal{C}_i| \mid \mathcal{C}_i \in \mathcal{C}(\lambda, r)\} = O(\log n)] = 1;$$

- if $\gamma = \lambda \pi r^2 > \gamma_c$, i.e., in the supercritical phase [19], there exists, w.h.p., exactly one *giant cluster* (giant component) $\mathcal{C}_i \in \mathcal{C}(\lambda, r)$ of size $|\mathcal{C}_i| = \Theta(n)$.

B. Distance to Giant Component

Connectivity is a necessary condition for a feasible routing scheme. From [32], [33], the connectivity of a routing scheme for homogeneous random networks $\mathcal{N}(\lambda, n)$ can be ensured when the maximum link length can reach $\Omega(\sqrt{\log n/\lambda})$. More

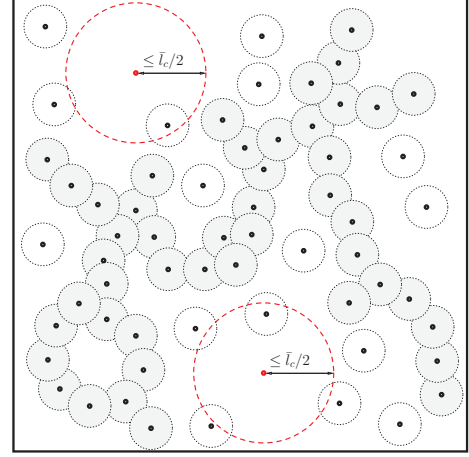


Fig. 4. Distance between an exterior node and the giant component.

specifically, by a geometric extension, we can obtain the following lemma based on Theorem 3.2 of [32].

Lemma 8: In Poisson Boolean model $\mathcal{B}(\lambda, r)$, with

$$\pi \cdot \lambda \cdot r^2 = \log n + \varsigma(n),$$

all disks with radius r are *connected* with probability 1 as $n \rightarrow \infty$ if and only if $\varsigma(n) \rightarrow \infty$.

From Lemma 8, we limit the nontrivial range of r in $[p_c/\sqrt{\lambda}, \sqrt{\log n/\lambda}]$, i.e., $r : [1/\sqrt{\lambda}, \sqrt{\log n/\lambda}]$. According to Lemma 7, in the Poisson Boolean model $\mathcal{B}(\lambda, r)$, there exists exactly one giant component, denoted by $\mathcal{C}(\lambda, r)$, with $|\mathcal{C}(\lambda, r)| = \Theta(n)$. Note that we take no account of the specific values of the involved constants, since they have no impact on the order of our final results.

In Poisson Boolean model $\mathcal{B}(\lambda, r)$, for any node outside the giant cluster $\mathcal{C}(\lambda, r)$, say an *exterior node* $u \notin \mathcal{C}(\lambda, r)$, we define the distance between u and the giant component by

$$\bar{l}_c(u) = \min_{v \in \mathcal{C}(\lambda, r)} |uv|.$$

Furthermore, we define the largest distance between exterior nodes and $\mathcal{C}(\lambda, r)$ as

$$\bar{l}_c^M[\mathcal{C}(\lambda, r)] := \max_{u \in \mathcal{V}(n) - \mathcal{C}(\lambda, r)} \bar{l}_c(u),$$

where $\mathcal{V}(n)$ denotes the set of all nodes in $\mathcal{N}(\lambda, n)$. Please see the illustration in Fig.4.

From Lemma 8, there is no node outside $\mathcal{C}(\lambda, r)$ when

$$\lambda \cdot r^2 = \frac{1}{\pi} \cdot (\log n + \varsigma(n)) \text{ if } \varsigma(n) \rightarrow \infty.$$

Then, we only consider the case that $\lambda \cdot r^2 = o(\log n)$, i.e., $r = o(\sqrt{\log n/\lambda})$. It holds that

$$\bar{l}_c^M[\mathcal{C}(\lambda, r)] > r, \text{ and } \bar{l}_c^M[\mathcal{C}(\lambda, r)] = o(\sqrt{\log n/\lambda}).$$

Next, we give a useful result for computing the upper bounds on network capacity.

Lemma 9: In Poisson Boolean model $\mathcal{B}(\lambda, r)$ with $r = o(\sqrt{\log n/\lambda})$, it holds that

$$\lambda \cdot r \cdot \bar{l}_c^M = \Omega(\log n), \text{ w.h.p.,} \quad (7)$$

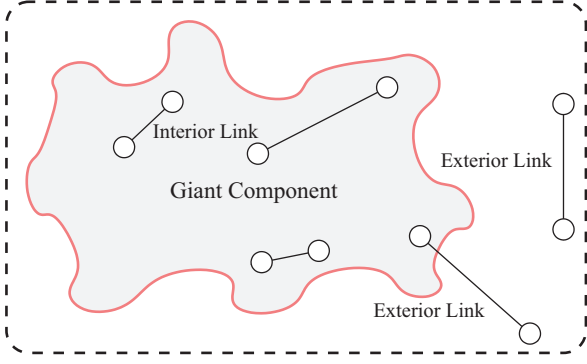


Fig. 5. Interior and Exterior Links.

where $\bar{l}_c^M := \bar{l}_c^M[C(\lambda, r)]$ for the sake of succinctness.

Prior to proving Lemma 9, we get the following lemma based on Corollary 1 of [34] by a geometric scaling method.

Lemma 10 ([34], [35]): For any exterior node, say $u \notin C(\lambda, r)$, it holds that for any $x \in [0, \sqrt{n/\lambda}]$,

$$\lim_{n \rightarrow \infty} \log \Pr[\bar{l}_c(u) > x] = - \lim_{n \rightarrow \infty} \varepsilon \cdot \lambda \cdot r \cdot x,$$

where $\varepsilon > 0$ is a constant.

Proof of Lemma 9: Firstly, we give a bound on the probability of event

$$\bar{E}(r): \lambda \cdot r \cdot \bar{l}_c^M = o(\log n) \text{ (a contradiction to Equation (7)).}$$

For any $u \notin C(\lambda, r)$, we define an event

$$\bar{E}_u(r): \lambda \cdot r \cdot \bar{l}_c(u) = o(\log n).$$

Then, it follows that

$$\Pr[\bar{E}(r)] = \Pr\left[\bigwedge_{u \notin C(\lambda, r)} \bar{E}_u(r)\right] \leq \left(1 - \frac{\varepsilon_1}{o(n)}\right)^{\varepsilon_2 n} \rightarrow 0,$$

where ε_1 and ε_2 are some constants. Hence, the lemma is proved. ■

VI. UPPER BOUNDS ON GENERAL MULTICAST CAPACITY

For any routing scheme, denote the maximum length (in the sense of order) of the links by l_c . According to [2], [10], in the networking-theoretic scaling laws [8], under the premise of ensuring routing connectivity, long-distance communication is not preferable, since the interference generated would preclude too many nodes from communicating. The optimal strategy is to confine to the nearest neighbor communication and maximize the number of simultaneous transmissions, *i.e.*, optimize the spatial reuse, [3]. From [32], [33], the routing connectivity of any scheme for homogeneous random networks can be ensured when the maximum link length is set to be $\Omega(\sqrt{\log n/\lambda})$. Then, we consider the range $l_c : [\mathfrak{p}_c/\sqrt{\lambda}, \sqrt{\log n/\lambda}]$, *i.e.*, $l_c : [1/\sqrt{\lambda}, \sqrt{\log n/\lambda}]$. From Lemma 7, in the Poisson Boolean model $\mathcal{B}(\lambda, l_c)$, there exists exactly one giant component, denoted by $\mathcal{C}(\lambda, l_c)$, with $|\mathcal{C}(\lambda, l_c)| = \Theta(n)$. Note that we take no account of the specific

values of the constants, for they have no impact on the order of our final results.

Then, the links of any multicast scheme can be divided into two classes as follows: A link is called an *interior link*, if both endpoints are located in $\mathcal{C}(\lambda, l_c)$; and it is called an *exterior link*, otherwise.

In the Poisson Boolean model $\mathcal{B}(\lambda, l_c)$, for any node outside the giant cluster $\mathcal{C}(\lambda, l_c)$, say $u \notin \mathcal{C}(\lambda, l_c)$, define the distance between u and the giant component by

$$\bar{l}_c(u) = \min_{v \in \mathcal{C}(\lambda, l_c)} |uv|.$$

Furthermore, we define

$$\bar{l}_c^M[C(\lambda, l_c)] := \max_{u \notin \mathcal{C}(\lambda, l_c/2)} \bar{l}_c(u).$$

Please see the illustration in Fig.4.

We derive the upper bounds on multicast capacity by considering two types of links comprehensively.

1) *Inside a Giant Component:* All links inside $\mathcal{C}(\lambda, l_c)$ have the length of $\Theta(l_c)$. The upper bound on capacity of these links can be computed as

$$\mathbf{R}_{l_c} = \min \left\{ 1, B \log \left(1 + \frac{l_c^{-\alpha}}{N_0} \right) \right\} = O(\min\{1, l_c^{-\alpha}\}).$$

Then, by combining with Lemma 3, we can obtain the following lemma.

Lemma 11: For any multicast scheme with the parameter l_c , the multicast throughput along the links inside $\mathcal{C}(\lambda, l_c)$ is

$$\text{at most of order } \Lambda_{l_c} = O \left(\frac{\min\{1, l_c^{-\alpha}\}}{\mathbf{L}(n_s, \frac{\sqrt{n}}{l_c \sqrt{n_d \lambda}})} \right).$$

Proof: According to Lemma 5, the length of any multicast tree is at least of order $\Omega(\sqrt{n_d n/\lambda})$. Then, for a given sender of any links inside the giant component, a multicast session passes through it with a probability of

$$\Omega \left(\min \left\{ 1, \frac{l_c \sqrt{n_d n/\lambda}}{n/\lambda} \right\} \right), \text{ i.e., } \Omega \left(\min \left\{ 1, \frac{l_c \sqrt{n_d \lambda}}{\sqrt{n}} \right\} \right).$$

By Lemma 3, the proof is completed. ■

2) *Outside a Giant Component:* Based on Lemma 9, we have,

Lemma 12: For any multicast scheme with l_c , the multicast throughput along the links between $\mathcal{C}(\lambda, l_c)$ and the nodes outside is at most of order $\Lambda_{\bar{l}_c^M} = O \left(\frac{\min\{1, (\frac{\lambda}{\log n})^{\alpha/2}\}}{\mathbf{L}(n_s, \frac{n \sqrt{\lambda} \cdot l_c}{n_d \sqrt{\log n}})} \right)$.

Proof: Since there must be a link outside the giant component with the length of $\sqrt{\log n/\lambda}$, the link capacity is bounded by

$$\begin{aligned} \mathbf{R}_{\bar{l}_c^M} &= \min \left\{ 1, B \log \left(1 + \frac{(\sqrt{\log n/\lambda})^{-\alpha}}{N_0} \right) \right\} \\ &= O \left(\min \left\{ 1, \left(\frac{\lambda}{\log n} \right)^{\alpha/2} \right\} \right). \end{aligned}$$

From Lemma 9, $\bar{l}_c^M = \Omega \left(\frac{\log n}{\lambda l_c} \right)$. It implies that $\bar{l}_c^M = \Omega(\sqrt{\log n/\lambda})$ because $l_c : [1/\sqrt{\lambda}, \sqrt{\log n/\lambda}]$. The probability

that a multicast session passes through such a link is of

$$\Omega \left(\min \left\{ 1, \frac{n_d \cdot \bar{l}_c^M \cdot \sqrt{\lambda}}{n \cdot \sqrt{\log n}} \right\} \right).$$

By Lemma 3, the proof is completed. ■

By combining Lemma 11 and Lemma 12, we finally obtain Theorem 1.

VII. CONCLUSION AND DISCUSSION

We derive the general upper bounds on the capacity for random wireless networks with a general node density. When the general results are specialized to the well-known random dense and extended networks, we show that our results close the open gaps between the upper and the lower bounds on the multicast capacity for both networks.

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Supplementary File

APPENDIX A

LOWER BOUNDS ON GENERAL MULTICAST CAPACITY

We design two general multicast schemes by using two types of hierarchical backbone systems in a well-integrated manner. One hierarchical backbone system consists of the *highways* and *ordinary arterial roads*; the other is composed of the *highways* and *parallel arterial roads*. Combining the achievable throughputs under our two schemes and other two schemes [36] that are respectively based only on ordinary arterial roads and parallel arterial roads, we derive the optimal throughput as the lower bounds on general multicast capacity according to different ranges of parameters.

For the sake of succinctness, we first introduce a notion called *scheme lattice* from [8].

Definition A.1 (Scheme Lattice, [8]): Divide the deployment region $\mathcal{R}(\lambda, n) = [0, \sqrt{n/\lambda}]^2$ into a lattice consisting of square cells of side length b , we call the lattice *scheme lattice* and denote it by $\mathbb{L}(\sqrt{n/\lambda}, b, \theta)$, where $\theta \in [0, \pi/4]$ is the minimum angle between the sides of the deployment region and produced cells.

In our multicast schemes, the backbones of routing comprise two levels: *highway system* and *arterial road system*. The highway system based on *bond percolation theory* [37] was originally proposed in [10]; and the connectivity-based arterial road system was devised in [36]. The main novelty of schemes in this work is the adoption of these two types of backbone systems in an integrated manner. For the sake of completeness, we introduce concisely the construction procedures of these backbone systems, and extend some relevant results in [10] and [36] into the scenarios with general node density by a geometric scaling, respectively.

A. Highway System

1) *Construction of highway system:* The highways are built based on scheme lattice $\mathbb{L}(\sqrt{n/\lambda}, \sqrt{c^2/\lambda}, \pi/4)$, as illustrated in Fig.A.1. Then, there are m^2 cells, where $m = \lceil \sqrt{n}/\sqrt{2c} \rceil^2$. A cell is non-empty (open) with the probability of $p \rightarrow 1 - \exp(-c^2)$, as $n \rightarrow \infty$, independently from each other. Based on $\mathbb{L}(\sqrt{n/\lambda}, \sqrt{c^2/\lambda}, \pi/4)$, draw a horizontal edge across half of the squares, and a vertical edge across the others, to obtain a new lattice as described in Fig.A.1. An edge h in the new lattice is *open* if the cell crossed by h is *open*, and call a path comprised of edges in the new lattice (Fig.A.1) *open* if it contains only open edges. Based on an open path penetrating the deployment region, as illustrated in Fig.A.1, we choose a node from each cell in $\mathbb{L}(\sqrt{n/\lambda}, \sqrt{c^2/\lambda}, \pi/4)$ corresponding to the edge of open path, call this node *highway-station*, connect a pair of highway-stations in two adjacent cells, and finally obtain a crossing path, and call it *highway*, as in Fig.A.1.

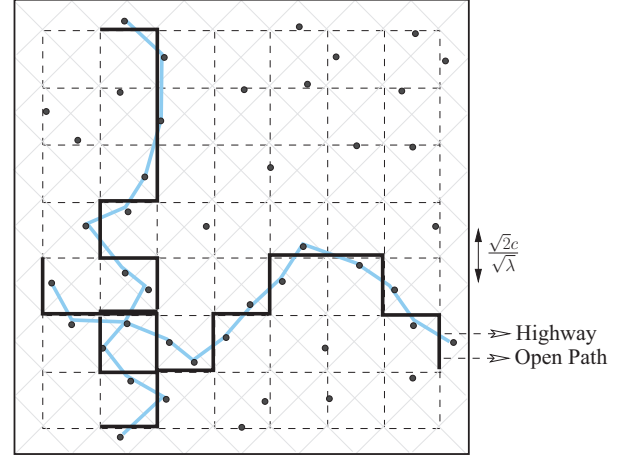


Fig. A.1. Building Horizontal and Vertical Highways, [1].

TABLE A.1
NOTATIONS USED IN THIS PAPER.

Notation	Meaning
$\mathbb{L}(\cdot, \cdot, \cdot)$	Scheme Lattice (Definition A.1)
AR	Arterial Road
AR-cell	The cell in $\mathbb{L}(\sqrt{n/\lambda}, 3\sqrt{\log n/\lambda}, 0)$
Station-cell	The square cell centered at AR-cell of area $\frac{4 \log n}{\lambda}$, Fig.A.2.
PA-cell	Parallel Assignment Cell-subsquare in AR-cell of area $\frac{9}{2\lambda}$.
O-AR, P-AR	Ordinary Arterial Road, Parallel Arterial Road
O-AP, P-AP	Ordinary Access Path, Parallel Access Path
\mathcal{U}_k	Spanning Set of Multicast Session \mathcal{M}_k
$\mathbf{S}_o(v)$	The entry point from node v to an assigned O-AR
$\mathbf{S}_p(v)$	The entry point from node v to an assigned P-AR
$\text{EST}(\mathcal{U}_k)$	An Euclidean Spanning Tree of Multicast Session \mathcal{M}_k
\mathbb{M}_o	Scheme based on only O-AR system
\mathbb{M}_p	Scheme based on only P-AR system
$\mathbb{M}_{o\&h}$	Scheme based on both O-AR and highway system
$\mathbb{M}_{p\&h}$	Scheme based on both P-AR and highway system

For a given constant $\kappa > 0$, partition the scheme lattice $\mathbb{L}(\sqrt{n/\lambda}, \sqrt{c^2/\lambda}, \pi/4)$ into horizontal (or vertical) rectangle slabs of size $m \times \kappa \log m$ (or $\kappa \log m \times m$), denoted by \mathcal{R}_i^H (or \mathcal{R}_i^V), where $m = \frac{\sqrt{n}}{\sqrt{2c}}$. Denote the number of disjoint horizontal (or vertical) highways within \mathcal{R}_i^H (or \mathcal{R}_i^V) by N_i^H (or N_i^V). The next lemma follows.

Lemma A.1: ([10]) For any κ and $p \in (5/6, 1)$ satisfying $2 + \kappa \log(6(1-p)) < 0$, there exists a $\eta = \eta(\kappa, p)$ such that

$$\lim_{m \rightarrow \infty} \Pr[N^h \geq \eta \log m] = 1, \quad \lim_{m \rightarrow \infty} \Pr[N^v \geq \eta \log m] = 1,$$

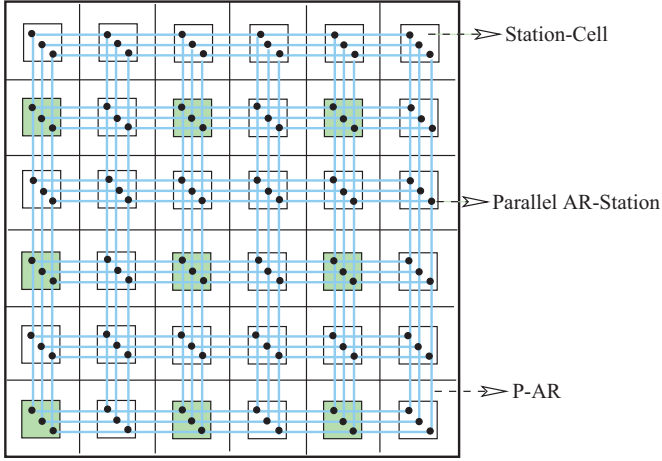


Fig. A.2. Parallel Arterial Roads. The shaded station-cells can be scheduled simultaneously. In any time slot, there are $2 \log n$ concurrent links initiated from every activated station-cell, [36].

where $N^H = \min_i N_i^H$ and $N^V = \min_i N_i^V$.

2) *Transmission scheduling for highway system*:: The highways can be scheduled by a 9-TDMA scheme based on scheme lattice $\mathbb{L}(\sqrt{n/\lambda}, \sqrt{c^2/\lambda}, \pi/4)$, [10]. By a similar to Theorem 3 in [10], we can prove that all highways can sustain w.h.p. the rate of order $\Omega(1)$.

B. Arterial Road (AR) System

We introduce two types of arterial road (AR) systems from [36]: *ordinary arterial road system* and *parallel arterial road system*, which perform better than the other according to the different density λ . Both AR systems are constructed based on the scheme lattice $\mathbb{L}(\sqrt{n/\lambda}, 3\sqrt{\log n/\lambda}, 0)$. Then, there are $\frac{n}{9 \log n}$ cells in $\mathbb{L}(\sqrt{n/\lambda}, 3\sqrt{\log n/\lambda}, 0)$, called *AR-cells*. Denote each row (or column) by $\tilde{\mathcal{R}}_i^h$ (or $\tilde{\mathcal{R}}_i^v$), where $i \in [1, \frac{\sqrt{n}}{3\sqrt{\log n}}]$. Then, from Lemma 4, for all $\frac{n}{9 \log n}$ AR-cells, the number of nodes is w.h.p. within $[\frac{9}{2} \log n, 18 \log n]$.

Firstly, we introduce the *ordinary arterial road system*.

1) *Ordinary Arterial Road System (O-AR system)*: The *ordinary arterial road system* can be obtained by choosing randomly one node from each cell, called *ordinary AR-station*, and connecting these stations in edge-adjacent cells. Then, we have

Lemma A.2 ([36]): By a 9-TDMA scheme, each ordinary arterial road in O-AR system can sustain a rate of

$$\mathbf{R}_{\text{O-AR}}(\lambda, n) = \begin{cases} \Theta\left(\frac{\lambda^{\frac{\alpha}{2}}}{(\log n)^{\frac{\alpha}{2}}}\right) & \text{when } \lambda : [1, \log n] \\ \Theta(1) & \text{when } \lambda : [\log n, n] \end{cases}$$

Next, we introduce the *parallel arterial road system*.

2) *Parallel Arterial Road System (P-AR system)*: In the center of each AR-cell, we set a smaller square of side length $2\sqrt{\log n/\lambda}$, as illustrated in Fig.A.2, call it *station-cell*. Then, by Lemma 4, we can prove that for all station-cells, there are, w.h.p., at least $2 \log n$ nodes.

The *horizontal* arterial roads in $\tilde{\mathcal{R}}_i^h$ is constructed by using the following operations: Firstly, for all $\frac{\sqrt{n}}{3\sqrt{\log n}}$ station-cells in $\tilde{\mathcal{R}}_i^h$, choose $2 \log n$ nodes from each station-cell, called *parallel AR-stations*. Secondly, connect those parallel AR-stations in the station-cells contained in the edge-adjacent AR-cells in a one-to-one pattern, as illustrated in Fig.A.2. In a similar way, we can construct the *vertical* arterial roads. We say that two arterial roads are *disjoint* if no station is shared by them. According to the procedure of construction above, there are $2 \log n$ disjoint horizontal (or vertical) arterial roads in every row (or column) of $\mathbb{L}(\sqrt{n/\lambda}, 3\sqrt{\log n/\lambda}, 0)$.

A 4-TDMA scheme, as depicted in Fig. A.2, is adopted to schedule arterial roads. The main technique called *parallel transmission scheduling* is: Instead of scheduling only one link in each activated station-cell (or cell) in each time slot, we consider scheduling $2 \log n$ links initiating from the same station-cell (or cell) together. It can be proven that this modification increases the total throughput for each cell by order of $\Theta(\log n)$, compared with only scheduling one link in each cell.

Lemma A.3 ([36]): Each P-AR can sustain a rate of

$$\mathbf{R}_{\text{P-AR}}(\lambda, n) = \begin{cases} \Theta\left(\frac{\lambda^{\frac{\alpha}{2}}}{(\log n)^{\frac{\alpha}{2}}}\right) & \text{when } \lambda : [1, (\log n)^{1-\frac{2}{\alpha}}] \\ \Theta\left(\frac{1}{\log n}\right) & \text{when } \lambda : [(\log n)^{1-\frac{2}{\alpha}}, n] \end{cases}$$

C. Access Paths

We assign nodes to the specific arterial roads by now. Next, we devise the *access path*, including *draining paths* and *delivering paths*, for every node to the arterial road system.

1) *Access Paths to O-AR System (O-APs)*: We call those links, along which the nodes outside drain the packets to O-AR system or the stations in O-AR system deliver the packets to the nodes outside, *ordinary access paths* (O-APs).

For every node outside ordinary arterial roads, say v , it drains (or receives) data packets to (or from) the ordinary AR-station in the AR-cell containing v , denoted by $\mathbf{S}_o(v)$, by a single hop called *ordinary draining path* (or *ordinary delivering path*).

A 4-TDMA scheme based on $\mathbb{L}(\sqrt{n/\lambda}, 3\sqrt{\log n/\lambda}, 0)$ is adopted to schedule the O-APs. Each slot can be further divided into $8 \log n$ subslots, ensuring that every link contained in each AR-cell can be scheduled once in a period of $4 \times 8 \log n$ subslots. Then, it follows that

Lemma A.4 ([36]): The rate of each ordinary access path, including ordinary draining path and ordinary delivering path, can also be sustained of

$$\mathbf{R}_{\text{O-AR}}(\lambda, n) = \begin{cases} \Theta\left(\frac{\lambda^{\frac{\alpha}{2}}}{(\log n)^{\frac{\alpha}{2}}}\right) & \text{when } \lambda : [1, \log n] \\ \Theta(1) & \text{when } \lambda : [\log n, n] \end{cases}$$

2) *Access Paths to P-AR System (P-APs)*: We call those links, along which the nodes outside drain the packets to P-AR system or the stations in P-AR system deliver the packets to the nodes outside, *parallel access paths* (P-APs).

For every node outside parallel arterial roads, say v , where $v \in \tilde{\mathcal{R}}_j^v$ and $v \in \tilde{\mathcal{R}}_i^h$, it drains the data packets into a parallel AR-station located in the adjacent AR-cell in $\tilde{\mathcal{R}}_j^v$, denoted by $\mathbf{S}_p(v)$, by a single hop called *parallel draining path* (Please see the illustration in Fig.A.3(a)); and receives the packets from the station, located in the adjacent AR-cell in $\tilde{\mathcal{R}}_i^h$, of a specific arterial road by a single hop called *parallel delivering path* (Please see the illustration in Fig.A.3(b)). Specifically, each AR-cell is further divided into $2 \log n$ subsquares, called *parallel assignment cell* (PA-cell), of area $\frac{9 \log n / \lambda}{2 \log n} = \frac{9}{2\lambda}$. Connect all nodes in the same PA-cell with the same P-AR station in the adjacent AR-cell to build the P-APs.

A 2-TDMA scheme is capable to schedule the draining paths (delivering paths, resp.) except those initiating from (terminating to, resp.) nodes in $\tilde{\mathcal{R}}_\delta^h$ ($\tilde{\mathcal{R}}_\delta^v$, resp.), where $\delta = \frac{\sqrt{n}}{3\sqrt{\log n}}$, and use an additional 1-TDMA scheme to schedule other draining paths (delivering paths, resp.). Please see the illustrations in Fig.A.3(a) and Fig.A.3(b). Then, it follows that

Lemma A.5 ([36]): The rate of each parallel access path, including parallel draining and parallel delivering paths, can also be sustained of $\mathbf{R}_{P-AR}(\lambda, n)$.

D. Multicast Routing Schemes

1) *Euclidean Spanning Tree:* We recall a result from [38].

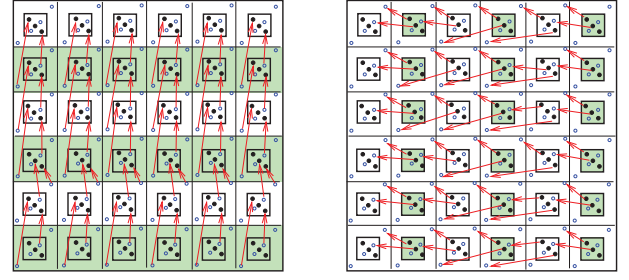
Lemma A.6 ([38]): For any spanning set \mathcal{U}_k consisting of $n_d + 1$ nodes placed in a square $\mathcal{R} = [0, a]^2$, the length of *Euclidean spanning tree* $\text{EST}(\mathcal{U}_k)$ obtained by the algorithm in [38] is at most of $2\sqrt{2} \cdot \sqrt{n_d + 1} \cdot a$.

Then, for any multicast session \mathcal{M}_k , based on its spanning set \mathcal{U}_k , we build an Euclidean spanning tree, denoted by $\text{EST}(\mathcal{U}_k)$. Denote the set of all edges of $\text{EST}(\mathcal{U}_k)$ by \mathcal{E}_k .

2) *Assignment of Backbones:* Now, we determine which backbones, including highway and AR, can be used by a specific communication-pair, i.e., a link $u \rightarrow v \in \mathcal{E}_k$.

Assignment of Arterial Roads: Denote the vertical O-AR (or P-AR) passing through the ordinary (or parallel) AR-station $\mathbf{S}_o(u)$ (or $\mathbf{S}_p(u)$) by $\mathbf{AR}_o^V(u)$ (or $\mathbf{AR}_p^V(u)$); and denote the horizontal O-AR (or P-AR) passing through the ordinary (or parallel) AR-station $\mathbf{S}_o(v)$ (or $\mathbf{S}_p(v)$) by $\mathbf{AR}_o^H(v)$ (or $\mathbf{AR}_p^H(v)$).

Assignment of Highways: Recall from Lemma A.1 that in each horizontal (or vertical) rectangle slab \mathcal{R}_i^H (or \mathcal{R}_i^V) of area $\sqrt{n} \times \kappa\sqrt{2c} \cdot \log \frac{\sqrt{n}}{\sqrt{2c}}$ (or $\kappa\sqrt{2c} \cdot \log \frac{\sqrt{n}}{\sqrt{2c}} \times \sqrt{n}$), there are at least $\eta \cdot \log \frac{\sqrt{n}}{\sqrt{2c}}$ horizontal (or vertical) highways. Divide further each horizontal (or vertical) slab into horizontal (or vertical) slice of area $\sqrt{n} \times \frac{\kappa\sqrt{2c}}{\eta}$ (or $\frac{\kappa\sqrt{2c}}{\eta} \times \sqrt{n}$). Choose any $\eta \cdot \log \frac{\sqrt{n}}{\sqrt{2c}}$ highways from each slab, and define an arbitrary bijection from those highways to the slices. For any node u located in a horizontal slice Slice_j^H (or vertical slice Slice_j^V), the packets initiating from u and terminating to v are assigned



(a) Parallel Draining Paths (b) Parallel Delivering Paths

Fig. A.3. (a) The shaded cells can be scheduled simultaneously. All draining paths except those initiating from nodes in \mathcal{R}_δ^h , where $\delta = \frac{\sqrt{n}}{3\sqrt{\log n}}$, can be scheduled once in $2 \times \frac{16 \log n}{2 \log n} = 16$ time slots. In each slot, $2 \log n$ links can be scheduled simultaneously. Here, $16 \log n$ is the maximum number of nodes in each cell, and $2 \log n$ is the number of stations in each cell. In addition, the nodes in \mathcal{R}_δ^h drain packets to the stations in $\mathcal{R}_{\delta-1}^h$, and those access paths can be scheduled by additional $\frac{16 \log n}{2 \log n} = 8$ time slots. (b) The shaded station-cells can be scheduled simultaneously. All delivering paths except those terminating to nodes in \mathcal{R}_δ^v , can be scheduled once in $2 \times \frac{16 \log n}{2 \log n} = 16$ time slots. In each slot, $2 \log n$ links can be scheduled simultaneously. In addition, the nodes in \mathcal{R}_δ^v receive packets from the stations in $\mathcal{R}_{\delta-1}^v$, and those access paths can be scheduled by additional $\frac{16 \log n}{2 \log n} = 8$ time slots.

to the horizontal highway $\mathbf{H}^H(u)$ and vertical highway $\mathbf{H}^V(v)$ that are mapped to the slices Slice_j^H and Slice_j^V , respectively.

3) *Multicast Routing Schemes:* For each multicast session \mathcal{M}_k with an Euclidean spanning tree $\text{EST}(\mathcal{U}_k)$, we build two types of multicast routing trees by two corresponding schemes, denoted by $\mathbb{M}_{o\&h}$ and $\mathbb{M}_{p\&h}$, as described in Table.A.1.

For each edge $u \rightarrow v \in \mathcal{E}_k$:

Under $\mathbb{M}_{o\&h}$, u drains the packets into the ordinary AR-station $\mathbf{S}_o(u)$ along a specific O-AP; the packets are transported along the vertical ordinary AR $\mathbf{AR}_o^V(u)$ to the assigned horizontal highway $\mathbf{H}^H(u)$; the packets are carried along $\mathbf{H}^H(u)$ and then the vertical highway $\mathbf{H}^V(v)$; the packets are transported along $\mathbf{AR}_o^H(v)$ to the ordinary AR-station $\mathbf{S}_o(v)$; and this station delivers the packets to v .

Under $\mathbb{M}_{p\&h}$, u drains the packets into the parallel AR-station $\mathbf{S}_p(u)$ along a specific P-AP; the packets are transported along the vertical parallel AR $\mathbf{AR}_p^V(u)$ to the assigned horizontal highway $\mathbf{H}^H(u)$; the packets are carried along $\mathbf{H}^H(u)$ and then the vertical highway $\mathbf{H}^V(v)$; the packets are transported along $\mathbf{AR}_p^H(v)$ to the parallel AR-station $\mathbf{S}_p(v)$; and this station delivers the packets to v .

When all links in \mathcal{E}_k are checked, merge the same edges (hops) and remove the circles that cannot break the connectivity of $\text{EST}(\mathcal{U}_k)$. Finally, we obtain the corresponding multicast routing trees.

E. Achievable Multicast Throughput

Let \mathbb{M}_o and \mathbb{M}_p denote respectively the schemes only using O-AR system and using P-AR system, [36]. By deriving the

optimal throughput based on these four schemes \mathbb{M}_o , \mathbb{M}_p , $\mathbb{M}_{o\&h}$, and $\mathbb{M}_{p\&h}$, we can obtain Lemma 1.

According to [36], under schemes \mathbb{M}_o and \mathbb{M}_p , the multicast throughputs can be respectively achieved as

$$\Lambda_o(\lambda, n) = \frac{\mathbf{R}_{O-AR}(\lambda, n)}{\mathbf{L}(n_s, \frac{1}{p_o})}, \quad \Lambda_p(\lambda, n) = \frac{\mathbf{R}_{P-AR}(\lambda, n)}{\mathbf{L}(n_s, \frac{1}{p_p})}.$$

Next, we analyze our new schemes, i.e., $\mathbb{M}_{o\&h}$ and $\mathbb{M}_{p\&h}$.

1) *Scheme Using Both the O-AR and Highway Systems, $\mathbb{M}_{o\&h}$* : The routing realization of any link in \mathcal{E}_k , say $u \rightarrow v$, can be divided into three phases: ordinary access path (O-AP) phase during which the packets are drained into O-ARs (or delivered from O-ARs) via O-APs, ordinary arterial Road (O-AR) phase during which the packets are drained into highways (or delivered from highways) along O-ARs, and highway phase during which the packets are transported along the highways. Consider the throughput during all three phases, we can obtain the multicast throughput under the scheme $\mathbb{M}_{o\&h}$ according to bottleneck principle.

Lemma A.7: Under the multicast scheme $\mathbb{M}_{o\&h}$, the multi-cast throughput is achieved as

$$\Lambda_{o\&h}(\lambda, n) = \min \left\{ \frac{\mathbf{R}_{O-AR}(\lambda, n)}{\mathbf{L}(n_s, \frac{1}{p_{oh, O-AR}})}, \frac{1}{\mathbf{L}(n_s, \frac{1}{p_{oh, H}})} \right\}.$$

Proof: Since O-APs can sustain the same rate (in order sense) as that of O-ARs, and the maximum burden of O-APs is necessarily not more than that of O-ARs, we neglect the analysis of O-AP phase, and only consider the O-AR phase and highway phase.

O-AR Phase: For any AR-station, say $\mathbf{S}_{oh, O-AR}$, define an event $E_k(\mathbf{S}_{oh, O-AR})$ for \mathcal{M}_k : \mathcal{M}_k shares the bandwidth of the link of an AR initiating from the station $\mathbf{S}_{oh, O-AR}$ during the O-AR phase of multicast scheme $\mathbb{M}_{o\&h}$. Clearly, if $E_k(\mathbf{S}_{oh, O-AR})$ happens, then there is an edge $u \rightarrow v \in \mathcal{E}_k$ such that the event $E_{k;u,v}(\mathbf{S}_{oh, O-AR})$ occurs, where the event $E_{k;u,v}(\mathbf{S}_{oh, O-AR})$ is defined as: the routing path of $u \rightarrow v$ under the scheme \mathbb{M}_o passes through $\mathbf{S}_{oh, O-AR}$. Obviously, $E_k(\mathbf{S}_{oh, O-AR}) = \bigcup_{uv \in \Pi_k} E_{k;u,v}(\mathbf{S}_{oh, O-AR})$. Then,

$$\begin{aligned} \Pr(E_k(\mathbf{S}_{oh, O-AR})) &\leq n_d \cdot \Pr(E_{k;u,v}(\mathbf{S}_{oh, O-AR})) \\ &\leq n_d \cdot \frac{6\sqrt{\log n/\lambda} \cdot \sqrt{2/\lambda c} \log \frac{\sqrt{n}}{\sqrt{2c}}}{n/\lambda} \\ &\leq \frac{6n_d \cdot (\log n)^{3/2}}{n} \end{aligned}$$

Define $p_{oh, O-AR} = \min\{\frac{6n_d \cdot (\log n)^{3/2}}{n}, 1\}$. Then, according to Lemma 3, we obtain that the throughput during the AR phase of scheme $\mathbb{M}_{o\&h}$ is achieved as $\frac{\mathbf{R}_{O-AR}(\lambda, n)}{\mathbf{L}(n_s, \frac{1}{p_{oh, O-AR}})}$.

Highway Phase: The routing realization of any multicast session \mathcal{M}_k passes through a station during the highway phase with the probability at most of

$$p_{oh, H} = \begin{cases} \Theta(\sqrt{\frac{n_d}{n}}) & \text{when } n_d : [1, \frac{n}{(\log n)^2}] \\ \Theta(\frac{n_d \log n}{n}) & \text{when } n_d : [\frac{n}{(\log n)^2}, \frac{n}{\log n}] \\ \Theta(1) & \text{when } n_d : [\frac{n}{\log n}, n] \end{cases}$$

From Lemma 3, we get that the throughput during highway phase of multicast scheme $\mathbb{M}_{o\&h}$ can be achieved as $\frac{\mathbf{R}_H(\lambda, n)}{\mathbf{L}(n_s, \frac{1}{p_{oh, H}})}$.

Multicast Throughput under Scheme $\mathbb{M}_{o\&h}$: According to *bottleneck principle*, we can obtain the final throughput under the scheme $\mathbb{M}_{o\&h}$. ■

2) *Scheme Using Both the P-AR and Highway Systems, $\mathbb{M}_{p\&h}$* : By a similar procedure to the analysis of $\mathbb{M}_{o\&h}$, we can obtain

Lemma A.8: Under the multicast scheme $\mathbb{M}_{o\&h}$, the multi-cast throughput is achieved as

$$\Lambda_{p\&h}(\lambda, n) = \min \left\{ \frac{\mathbf{R}_{P-AR}(\lambda, n)}{\mathbf{L}(n_s, \frac{1}{p_{ph, P-AR}})}, \frac{1}{\mathbf{L}(n_s, \frac{1}{p_{ph, H}})} \right\}.$$